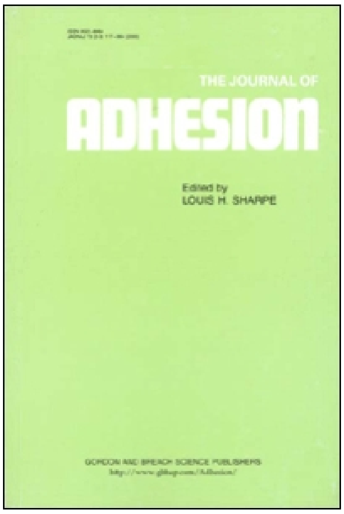

The Journal of Adhesion
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453635

On the Detachment of a Rigid Inclusion from an Elastic Matrix

D. W. Nicholson

To cite this Article Nicholson, D. W.(1979) 'On the Detachment of a Rigid Inclusion from an Elastic Matrix', The Journal of Adhesion, 10: 3, 255 — 260

To link to this Article: DOI: 10.1080/00218467908544628

URL: http://dx.doi.org/10.1080/00218467908544628

J. Adhesion, 1979, Vol. 10, pp. 255–260
0021-8464/79/1003-0255 $04.50/0

On the Detachment of a Rigid Inclusion from an Elastic Matrix

D. W. NICHOLSON

Explosion Effects Branch, Naval Surface Weapons Center, White Oak, Silver Spring, Maryland 20910, U.S.A.

(*Received January 30, 1979; in Final Form June 27, 1979*)

In many materials, such as ductile metals and some polymers, nucleation of voids at weakly adhered inclusions and grains can initiate the fracture process. Accordingly, we consider a rigid spherical inclusion at the center of, and completely adhered to, a much larger sphere of linearly elastic isotropic material. The adhesive bond is assumed to be weak, and the matrix sphere is subject to uniform fixed radial stress on its outer surface. The criterion of detachment is investigated. Our approach follows that of the classical Griffith analysis for the centrally cracked plate, and our results appear to be new. In particular, a relation is derived giving the critical inclusion radius at which detachment is predicted to occur under a given stress. For example, this radius is found to depend inversely on the square of the applied stress.

INTRODUCTION

Many materials contain dispersed particles and grains which can become detached under suitable conditions. For example, in ductile metals such as mild steel and commercially pure titanium, fracture can begin as the nucleation of microvoids around inclusions and grains under high hydrostatic tension. The microvoids then grow under increasing stress, and finally coalesce to form cracks. In polymers many materials have fillers, for example, carbon black in rubber. Under typical dispersion conditions, fillers can aggregate into relatively large particles, which then detach and thereby serve as sites for tear initiation.

Of course, the stress analysis of inclusions has been extensively investigated in the past.[1] But it appears that detachment has been given considerably less attention.[2] Here we derive a detachment criterion using the approach established in Griffith's classical treatment[3] of spontaneous growth of a central crack in a large plate. Our results appear to be new.

To be sure, some attempts have been made previously to use a Griffith approach. In Ref. 4, the energy released from the elastic inclusion was equated with the work of detachment. The energy released from the elastic strain energy field of the matrix was disregarded. Also neglected was the energy contribution associated with the matrix boundary conditions; for example, the work of fixed external stresses on the external boundary displacements arising from detachment. The simplifications in Ref. 4 cause underestimation of the energy available for detachment, and in particular, exclude detachment of rigid inclusions with finite adhesion.

In Ref. 5, an analysis was reported including the effect of energy release from the matrix. But the contribution associated with the boundary condition was still neglected and the energy available for detachment was still underestimated.

In Ref. 2, the treatment in Ref. 5 was claimed to predict unrealistically small critical inclusion sizes for detachment (under the initial yield stress in a metal). The energy criterion was subsequently described as necessary but not sufficient, and a strength model was proposed for the interface. However, the predictions in Ref. 5 were to some extent supported by descriptions of experiments showing very small detached particles at moderate plastic strains.

In any event, critical inclusion size predictions presuppose knowledge of adhesion energies, a very difficult quantity to determine. (Very tenuous formulae were used in Ref. 5 to obtain estimates.) It would seem more promising to estimate adhesion energies from the critical inclusion sizes. To do so, a theoretical relation is needed, and is furnished in the present work.

CRITICAL INCLUSION SIZE RELATION

We consider a rigid spherical inclusion of radius r_0 which is centered in a large sphere of radius r_∞. The matrix sphere experiences small strains and obeys Hooke's law with elastic modulus E and Poisson's ratio ν. A uniform tensile radial stress p is imposed at r_∞, and is fixed. The inclusion is initially completely bonded to the matrix, so that there are no detached zones at the interface. We assume that the adhesion is poor. Specifically, the bond is characterized by a value of γ, the energy absorbed in debonding a unit area of the interface, and this value is lower than the corresponding cohesive fracture energy.

In the paragraphs to follow, we will derive the critical radius r^* for detachment under fixed stress p as

$$r_0 \geqslant r^* = \frac{\gamma E}{3p^2}\frac{4}{3}\frac{1+\nu}{(1-\nu)^2} \qquad (1)$$

The critical inclusion radius is thus inversely proportional to the square of the applied stress.

ANALYSIS

The energies controlling detachment

Initially there is complete bonding. But suppose that detachment suddenly occurs everywhere along the interface. The elastic strain energy of the matrix sphere is changed, say by U. Further, work W may be done by p in bringing about the displacement occurring at r_∞ because of the detachment. But energy Γ is absorbed in destroying the bond.

The detachment is predicted to occur if

$$W > U + \Gamma. \qquad (2)$$

In words, detachment occurs if sufficient energy is thereby made available.

Essentially, the same principle has been applied by Griffith[3] to predict spontaneous growth of a central crack in a large elastic/brittle plate under (remote) tensile stress. Failure is predicted to occur at a particular crack length determined by the applied stress. At the critical condition, subsequent crack growth will release more energy than will be absorbed in generating new crack surface area.

Our application involves a slight variation on Griffith's approach. In the crack problem it is possible to compare the energies of incrementally different (virtual) equilibrium configurations, corresponding to incremental increases in the crack length. But in the present treatment, the energy difference between the completely attached and completely detached configurations is finite. We have tacitly assumed that all configurations of partial detachment are unstable if (1) is satisfied. We expect this assumption to be particularly appropriate when the inclusions are small: $r_0 \ll r_\infty$.

Basic elastic relations

Let u represent the radial displacement, σ_r the radial stress, and σ_θ the tangential stress. Under radially symmetric stress p at r_∞, the solution is[6]

$$u = \alpha r + \beta/r^2 \qquad (3a)$$

$$\sigma_r = \alpha/\mu_2 - 2\beta/\mu_1 r^3 \qquad (3b)$$

$$\sigma_\theta = \alpha/\mu_2 + \beta/\mu_1 r^3 \qquad (3c)$$

where, in conventional notation,

$$\mu_1 = (1+\nu)/E \qquad \mu_2 = (1-2\nu)/E$$

and α and β are constants of integration. The strain energy density is readily shown to be

$$w_e = \frac{\mu_2}{6}(\sigma_r+2\sigma_\theta)^2+\frac{2\mu_1}{6}(\sigma_r-\sigma_\theta)^2$$
$$= \frac{3}{2}\frac{\alpha^2}{\mu_2}+\frac{3\beta^2}{\mu_1}\frac{1}{r^6}.$$

The total strain energy in the matrix sphere is then

$$U_e = \int_{r_0}^{r_\infty} w_e 4\pi r^2\,dr$$
$$= \frac{2\pi}{\mu_2}\alpha^2(r_\infty^3-r_0^3)+\frac{4\pi}{\mu_1}\beta^2\{1/r_0^3-1/r_\infty^3\}. \tag{4}$$

We assume that the rigid inclusion stores no strain energy.

Elastic solutions

i) *Still-attached configuration*

The appropriate boundary conditions are

$$u = 0 \quad \text{when} \quad r = r_0 \tag{5a}$$
$$\sigma_r = p \quad \text{when} \quad r = r_\infty \tag{5b}$$

It readily follows that

$$\alpha = p\mu_2/\{1+2\mu_2 r_0^3/\mu_1 r_\infty^3\}$$
$$\beta = -p\mu_2 r_0^3/\{1+2\mu_2 r_0^3/\mu_1 r_\infty^3\}$$

and hence

$$U_1 \equiv U_e = 2\pi p^2\mu_2(r_\infty^3-r_0^3)/\{1+2\mu_2 r_0^3/\mu_1 r_\infty^3\}. \tag{6}$$

The displacement at r_∞ is

$$u_1 \equiv u = r_\infty p\mu_2[1-r_0^3/r_\infty^3]/\{1+2\mu_2 r_0^3/\mu_1 r_\infty^3\} \tag{7}$$

ii) *Detached configuration*

Now we have that

$$\sigma_r = 0 \quad \text{when} \quad r = r_0 \tag{8a}$$
$$\sigma_r = p \quad \text{when} \quad r = r_\infty \tag{8b}$$

and

$$\alpha = \mu_2 p/\{1-r_0^3/r_\infty^3\}$$
$$\beta = \tfrac{1}{2}\mu_1 p r_0^3 r_\infty^3/(r_\infty^3-r_0^3).$$

It readily follows that

$$U_2 \equiv U_e = 2\mu_2 p^2 \pi r_\infty^6 [1 + \tfrac{1}{2}\mu_1 r_0^3/\mu_2 r_\infty^3] \tag{9}$$

Also, at r_∞

$$u_2 \equiv u = r_\infty p \mu_2 \{1 + \tfrac{1}{2}\mu_1 r_0^3/\mu_2 r_\infty^3\}/\{1 - r_0^3/r_\infty^3\}. \tag{10}$$

Detachment criterion

The strain energy increase with detachment is $U_2 - U_1$. Taking the limit in (6, 9) as r_∞ approaches infinity, we may derive that

$$\lim_{r_\infty \to \infty} (U_2 - U_1) = 2\pi \mu_2 p^2 r_0^3 [2 + \tfrac{1}{2}\mu_1/\mu_2 + 2\mu_2/\mu_1] \tag{11}$$

The work is

$$W = 4\pi p r_\infty^3 (u_2 - u_1).$$

Substituting (7, 10) and taking the limit we find

$$\lim_{r_\infty \to \infty} W = 4\pi p^2 \mu_2 r_0^3 [2 + \tfrac{1}{2}\mu_1/\mu_2 + 2\mu_2/\mu_1]. \tag{12}$$

But energy Γ is absorbed in bond failure, where

$$\Gamma = 4\pi r_0^2 \gamma \tag{13}$$

Substituting (11–13) into (2) gives the detachment criterion as

$$r_0 \geqslant r^* = \frac{\gamma E}{3p^2} \frac{4}{3} \frac{1+\nu}{(1-\nu)^2} \tag{14}$$

as asserted in (1).

Equation (14) leads to several interesting conclusions. First, the critical inclusion radius r^* increases linearly with the modulus E of the matrix. This is certainly expected, since in this event the elastic properties of matrix and inclusion are becoming closer.

Secondly, r^* grows linearly with γ, the measure of the quality of the bond. Consequently, increased inclusion size is equivalent to poorer adhesion as far as detachment is concerned.

Thirdly, r^* increases nearly linearly with ν, which measures the resistance of the matrix material to compression. The r_c value for an incompressible body ($\nu = \tfrac{1}{2}$) is six times that for a body which cannot resist compression ($\nu = 0$). To the present author, it is surprising that the effect of compressibility is not greater.

Finally, r^* is inversely proportional to the square of the applied pressure. So, for example, a two-fold increase in inclusion size reduces the critical stress for detachment by a factor of four.

CONCLUSION

A model problem has been treated regarding the detachment of a centrally located rigid spherical inclusion from a large elastic matrix sphere under fixed radial tensile stress p. The approach used follows the Griffith analysis for the center-cracked plate. A relation has been derived for critical inclusion radius r^*. One prediction is that r^* is inversely proportional to p^2.

References

1. E. Sternberg, R. A. Eubanks and M. A. Sadowsky, *Proc. 1st U.S. National Cong. Appl. Mech.*, 209 (1951).
2. A. S. Argon, J. Im and R. Safoglu, *Metall. Trans. A*. **6**, 825 (1975).
3. A. A. Griffith, *Phil. Trans A* **221**, 1963 (1921).
4. J. Gurland and J. Plateau, *Trans. ASM* **56**, 442 (1963).
5. K. Tanaka, T. Mori, T. Nakamura, *Phil. Mag*. **21**, 267 (1970).
6. R. Hill, *The Mathematical Theory of Plasticity* (Oxford, The Clarendon Press, London, 1950).